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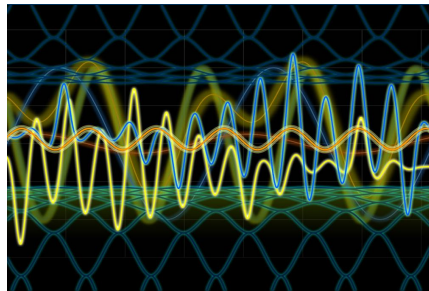
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Envelope Functions for Sound Spectra of Pipe Organ Ranks and the Influence of Pitch on Tonal Timbre

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Long-term average sound spectra (LTAS) of organ pipes from different flue and reed ranks have been studied with the intention to obtain pitch-dependent LTAS models for the constant part of the pipe sound. The measured LTAS of each rank of pipes cover up to nine octaves within the tonal range $C_0 \dots C_9$. The envelope functions are piecewise linearly parametrized and serve as empirical approximations for the sound pressure levels of the harmonic partials. They allow for extrapolating the pitch range of most ranks and thus to explore these spectra beyond their physically existing compass. Two parameters for tonal timbre were calculated from the spectra: the first is a weighted average slope of the envelope function; the second is the spectral centroid of the sound spectrum. Each rank of pipes depicts as characteristic curve (from bass to treble) in a two-dimensional timbre chart of these two parameters, in which the different families of organ tone clearly separate from each other. The use of such timbre charts could assist in the work of pipe voicing by means of an electronic voicing device, which extracts the LTAS from the measured pipe sound and calculates the timbre chart from it.

1. INTRODUCTION

The pipe organ is a wind instrument with one sound generator for each tone and timbre. A set of organ pipes across the key compass designed to generate the same tone quality is called rank. Among many other parameters, the dimensions of the pipe (including all their components) influence the sound spectrum of a pipe. See Fletcher's book for a treatise of the various aspects of the physics of different types of organ pipes¹.

This article investigates the sound spectra of different organ pipes excluding the initial speech onset and the tone release. The sound spectra of the middle part of pipe organ tones are approximately constant, because any existing sound fluctuations are levelled in the long-term average sound spectra (LTAS). For this reason, the analysis of LTAS allows to evaluate the sound of the different organs by playing the same tones with equal rank combinations² and to compare whole ranks by analyzing full-tone scales throughout the compass of a rank³, but is also suitable for the detailed analysis of single pipes (see references in Table 1). The tone of a single pipe depends on so many parameters (the recording position, the wind pressure, etc.) that the calculation of its sound spectrum, even when based on sophisticated physical models (see e.g. here^{4,5,6}), still remains an approximation. Furthermore, the seasonal temperature can change the sound of an organ noticeably⁷. When referring to the sound of a rank at certain pitch, one must keep in mind that the names of organ ranks do not precisely indicate how its pipes sound, because many variants of identically named organ ranks and the way they are voiced exist. The same problem occurs, when describing the sound of a pipe with words and agreeing on a common understanding of verbal descriptors^{8,9}. Thus, a parameter set based on numbers, which corresponds to objective and measurable data, can become a helpful classification scheme for organ ranks.

This work approximates the LTAS from 35 different organ ranks with empirically derived envelope functions over wide pitch ranges. These envelopes describe the sound pressure of each partial of the complex tone. The idea is to keep the number of the parameters in the model small, because otherwise one had to exclude all sound spectra having only a few measurable partials (e.g. high-pitched FLUTES). The envelope function depends on pitch and compromises simplicity and complexity (see Chapter 3).

The typical patterns in the envelope functions of different organ pipes and ranks of pipes yielded the graphical representation of two timbre parameters, which are useful to classify the tonal timbre of organ pipes.

2. SOUND SPECTRA

The empirical envelope function model has been developed and tested using 81 published sound spectra from single organ pipes (Table 1). Twelve of these data sets serve as reference points for Fig. 4 and as representative sound spectra for different families of organ tone throughout this study.

Table 1. Pitch ranges for the 81 sound spectra from literature including references.

Rank	Pitch or pitch range
TIBIA (stopped)	C ₄ ¹⁰
STOPPED FLUTE, GEDACKT	C ₂ ^{11,12} , G ₂ ¹³ , C ₃ ¹² , C ₄ ^{10,12} , C ₄ ...C ₆ (three organ builders) ¹⁴ , C ₅ ¹²
QUINTADENA	C ₂ ...B ₂ ¹⁵ , C ₅ ...B ₅ ¹⁵
CHIMNEY FLUTE	C ₃ (three spectra) ¹⁶
CONICAL FLUTE	B ₂ , C ₃ (two mouths), G ₃ , C ₄ ¹¹
OPEN FLUTE; PICCOLO	unknown pitch ¹⁶ ; C ₆ , C ₇ , C ₈ , C ₉ ¹¹
NACHTHORN	C ₄ * ¹⁵
OPEN DIAPASON, PRINZIPAL; OKTAVE, QUINTE	unknown pitch ¹⁶ , C ₂ ...B ₂ ¹⁵ , G# ₂ ¹³ , A ₂ ¹⁹ , C ₃ ¹² , C ₄ * ¹⁷ , C ₄ * ¹⁸ , C ₄ ¹² , C ₅ ^{11,12} , C ₅ ...B ₅ ¹⁵ ; C ₄ , C ₅ ¹² , G ₄ ¹²
GEIGEN DIAPASON	C ₄ ¹⁸ (four organ builders, spectrum A*)
SALICIONAL; ZARTGEIGE	unknown pitch ¹⁶ , C ₄ ¹⁸ (four organ builders, spectrum A*); C ₄ * ¹⁷
VIOL D'ORCHESTRE	C ₄ ¹⁸ (four organ builders, spectrum A*)
TROMPETE	C ₂ ...B ₂ ¹⁵ , C ₄ ¹⁸ (four organ builders, spectrum B*), C ₅ ...B ₅ ¹⁵
CORNOPEAN	C ₄ ¹⁸ (four organ builders, spectrum X*)

FAGOTT	E ₂ , B ₃ , E ₅ ²⁰
KRUMMHORN	F# ₄ * ²¹
PHYSHARMONIKA (free reed)	C ₃ ²²
COR ANGLAIS (free reed)	C ₄ ²¹
KLARINETTE (free reed)	F# ₄ * ²¹ (two organ builders), C ₄ ²³ (four spectra, 1/2 length resonator*)
Explanations: Averaged spectra within the given pitch range. *: Reference data points used in Fig. 4	

After the model for the spectral envelopes worked for these sound spectra, it was desirable to test it at ranks of pipes across their complete pitch range. For this purpose, audio files were generated from digitally recording ranks from organ sample sets with the software Hauptwerk²⁴. The first instrument, natively included as sample set in the Hauptwerk software, is the Brindley & Foster organ in St. Anne's, Moseley, UK, with 30 ranks, most of them installed in 1907²⁵. The sample set has been recorded with 96 kHz and 32 bit resolution in 2009. The second sample set is the "Paramount 450", a virtual theatre organ²⁶ containing 52 ranks, most of them built by the Wurlitzer Organ Co. in the 1920's. The selected ranks of these two instruments are tabulated in Table 2 and Table 3. In the following, the sample sets are referred to as "StA" and "P450". Compared to church organs (like StA) theatre organs operate on higher wind pressure facilitating to enhance the range of tonal timbre by pipe voicing. The ten ranks chosen from P450 shall complement with the 25 from the StA instrument.

Table 2. The 25 selected ranks of the StA sample set and their tonal range.

Rank and Footage (Division)	Pitch range	Rank and Footage (Division)	Pitch range
LIEBLICH BOURDON 16' (Sw)	C ₁ ...C ₆	LIEBLICH GEDACT 8' (Gt)	C ₂ ...C ₇
ROHR FLUTE 8' (Sw)	C ₂ ...C ₇	DULCIANA 8' (Gt)	C ₂ ...C ₇
GEIGEN PRINCIPAL 8' (Sw)	C ₂ ...C ₇	OPEN DIAPASON LARGE 8' (Gt)	C ₂ ...C ₇
VIOLA DI GAMBA 8' (Sw)	C ₂ ...C ₇	OPEN DIAPASON SMALL 8' (Gt)	C ₂ ...C ₇
VOIX CELESTE 8' (Sw)	C ₂ ...C ₇	CLEAR FLUTE 4' (Gt)	C ₃ ...C ₈
FLAUTO MAGICO 4' (Sw)	C ₃ ...C ₈	PRINCIPAL 4' (Gt)	C ₃ ...C ₈
SALICET 4' (Sw)	C ₃ ...C ₈	FIFTEENTH 2' (Gt)	C ₄ ...C ₉
PICCOLO 2' (Sw)	C ₄ ...C ₉	TRUMPET 8' - CLARION 4' (Gt)	C ₂ ...C ₈
CONTRA FAGOTTO 16' (Sw)	C ₁ ...C ₆	ECHO BASS 16' (Ped)	C ₁ ...F ₃
CLARINET 8' (Sw)	C ₂ ...C ₇	SUB-BASS 16' (Ped)	C ₁ ...F ₃
TRUMPET 8' (Sw)	C ₂ ...C ₇	BASS FLUTE 8' (Ped)	C ₂ ...F ₄
OBOE 8' (Sw)	C ₂ ...C ₇	OPEN DIAPASON 16', PRINCIPAL 8', FIFTEENTH 4' (Ped)	C ₁ ...F ₅
TROMBONE 16' (Sw)	C ₁ ...F ₃		
All ranks except for the Mixtures from this sample set have been selected for analysis.			

Table 3. The ten selected ranks of the P450 sample set and their tonal range.

Rank names	Division	Pitch range
TIBIA (CLAUSA) 16'-8', PICCOLO (TIBIA) 4'-2'	Main	C ₁ ...C ₉
CONTRA BOURDON 32', BOURDON 16', CONCERT FLUTE 8', FLUTE 4', PICCOLO 2'	Main	C ₀ ...C ₉
HARMONIC FLUTE 4'	Solo	C ₃ ...C ₈
CONTRA DIAPHONE 32', DIAPHONE 16', DIAPHONIC DIAPASON 8', OCTAVE 4'	Main	C ₀ ...C ₈
GAMBA 16'-8', GAMBETTE 4' (without CELESTE)	Solo	C ₁ ...C ₈
CONTRA SALICIONAL 32', SALICIONAL 16'-8', SALICETTE 4' (without CELESTE)	Main	C ₀ ...C ₈
CLARINET 16'-8'	Main	C ₁ ...C ₇
SOLO ENGLISH HORN 16'-8'	Solo	C ₁ ...C ₇
CONTRA BOMBARDE 32', BOMBARDE 16', TUBA MIRABILIS 8'	Solo	C ₀ ...C ₇
OPHICLEIDE 16', TUBA HORN 8'	Main	C ₁ ...C ₇

The ranks listed in Table 2 and Table 3 were loaded as sample sets StA and P450 in the software Hauptwerk. The tremulants and separately recorded wind noise were both deactivated (P450) and no additional convolution reverb was added when playing the sounds of the recorded organ pipes. Three tones per octave, spaced in intervals of major thirds ($C_0, E_0, G\#_0, \dots, C_9$), were chosen to generate WAV files (stereo, 48 kHz sampling rate, 24 bit resolution). Sections of at least 10...15 s duration were used to generate the LTAS with the software Audacity²⁷. The Fourier transform used a Hann window to obtain the sound pressure levels (SPL) at the partials above the noise threshold, which were then manually extracted to generate simplified harmonic spectra (as sketched in Fig. 1). The pitch-dependent window size was set to 2^{15-k} for any pitch $C_k, E_k, G\#_k$ in the octaves $k = 0 \dots 8$ and set to 2^7 for C_9 . SPL L_n were extracted for partial numbers $n \leq 20$ up to that partial number n , for which the background noise dominated. This process might be automated using pattern recognition²⁸.

3. SOUND SPECTRA EVALUATION

This chapter describes how the sound spectra of the pipes and ranks listed in Table 1, Table 2 and Table 3 have been approximated based on a piecewise linear envelope function (Fig. 1). Their categorization using tonal timbre parameters was performed with the spectral centroid and the average weighted slope of the modelled LTAS. Of course, one may calculate these timbre parameters from the LTAS directly without modelling them.

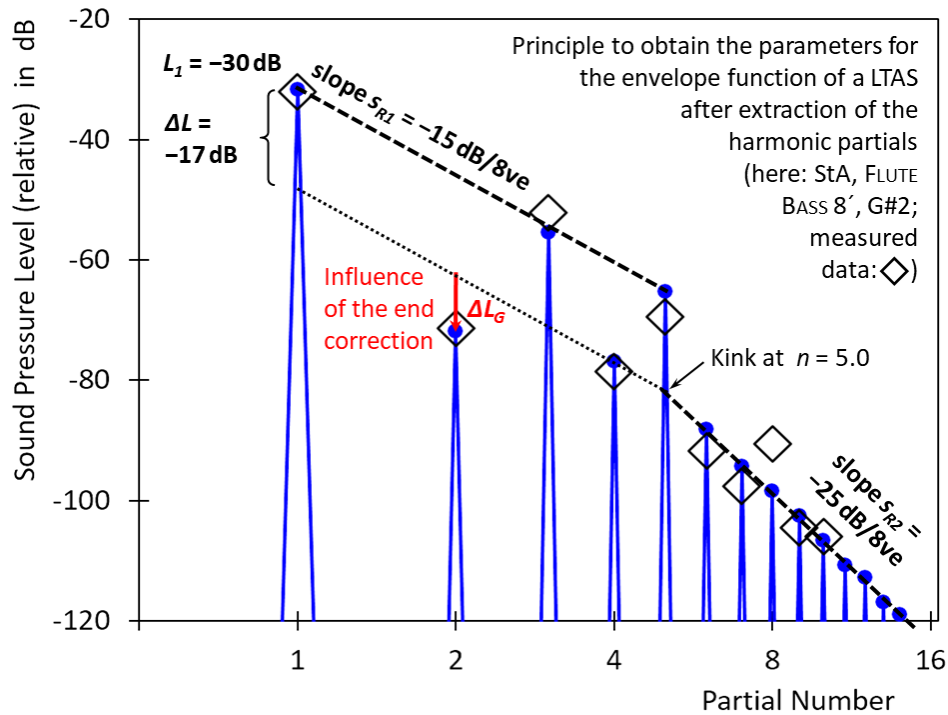


Figure 1. Illustration of the principle of the straight-line approach to approximate the spectral envelope.

A. ENVELOPE FUNCTION MODEL

The sound produced by an organ pipe consists of three successive parts: it begins with the attack transient (when the sound stabilizes after the wind inlet into the pipe has begun) followed by a relatively stable steady sound thereafter and, finally, there is a sound release when the wind supply stops. Here, we only concentrate on the sound spectra of the middle part, of which we consider the time-average (LTAS).

The LTAS of organ pipes contain several components, ordered by their typical contribution strength: the first three are the spectrum of the harmonic partials, additional broader peaks (the eigenfrequencies of the resonator tube, which generally do not coincide with the harmonic partials), and a frequency-dependent noise background^{16,17}. In addition, not harmonically related sharp peaks may be detectable²⁹. Here, we exclusively consider the first (and usually strongest) contribution of the harmonic partials, whose frequencies f_n relate to the frequency of the fundamental f_1 ($n = 1$) as:

$$f_n = n \cdot f_1, \text{ where } n \in \mathbb{N} \text{ is the number of the } n\text{-th partial} \quad (1)$$

Dividing Eq. (1) by the fundamental frequency, f_1 , yields n and allows comparing the SPL L_n of sound spectra of organ pipes with different fundamental frequencies f_1 directly. Dividing the sound spectra into a harmonic and an inharmonic part yields:

$$L_n = \begin{cases} L_n & , \text{ if } n \in \mathbb{N} \\ L_0(f) & , \text{ else} \end{cases} \quad (2)$$

with the SPL of the n -th partial L_n and the frequency-dependent background level $L_0(f)$. For simplicity $L_0(f)$ is set here to a large and constant negative value, e.g. $L_0 = -200$ dB, hence ignoring any inharmonic contributions including background noise.

The course of the envelope fitted to the SPL of the harmonic partials, L_n , correlates to the perceived sound of organ pipes¹⁸ and polynomials up to the fifth order have been used to approximate recorded spectra¹⁵. The disadvantage of higher-order polynomials is, however, that the values of their coefficients are not easily to interpret when comparing different sound spectra by them without visualizing the whole polynomial function.

For some types of wind-driven instruments, the shape of the LTAS is known in principle. Fletcher's book¹ contains examples for the reed-woodwind instruments Clarinet and Saxophone, whose spectra are confined by two slopes with a smooth transition at a certain frequency. Pykett has proposed to plot the harmonic LTAS of organ pipes from various ranks over the logarithm of the partial number n , to then approach them with piecewise linear envelope functions^{30,31,32}. Choosing logarithmic scaling of the abscissa, say $\log_2(n)$, makes the distance of any dyad, e.g. an octave, constant along this axis. This facilitates to express the values for the slopes of the straight lines describing the SPL in spectral range 1 and 2 as s_{R1} and s_{R2} , respectively, as depicted in Fig. 1.

It is comfortable to introduce a quantity measuring the pitch distance Δp between the fundamental (at the frequency f_1) and the n -th partial in semitones, or – alternatively – in octaves, Δo :

$$\Delta o = \Delta p / 12 = (p_n - p_1) / 12 \quad (3)$$

In this formula, pitch is understood as pitch category p , an integer number counting the semitone steps beginning at C_0 ; for example $p(C_4) = 48$. The distances Δo and Δp relate to the partial number n as follows:

$$n = \lceil 2^{\Delta o} \rceil \text{ and } \Delta p = \lceil 12 \cdot \log_2(n) \rceil, \text{ where } \lceil \dots \rceil \text{ means rounding to the nearest integer.} \quad (4)$$

Certain shapes of organ pipe resonators (e.g. stopped flue pipes or cylindrical reed pipes) affect the SPL of the even-numbered partials¹. The envelope model function takes into account for this by adding a (usually negative) level ΔL to the even-numbered partials, if $n \leq n_{ip}$. During testing the model on measured sound spectra it became obvious that ΔL should be applied all even and odd partials, if $n > n_{ip}$ (Fig. 1), because this resulted in better agreement with the data. Note that n_{ip} does not need to be an integer number or – in other words – the position of kink separating the two slope ranges in Fig. 1 may be located anywhere as long as $n_{ip} \geq 2$.

The SPL of the partials $n \leq n_{ip}$, is confined by the slope s_{R1} and an eventual shift ΔL :

$$L_n(n \leq n_{ip}) = \begin{cases} L_1 + s_{R1} \cdot \Delta o & , \text{ if } n \text{ is odd} \\ \Delta L + L_1 + s_{R1} \cdot \Delta o & , \text{ if } n \text{ is even} \end{cases} \quad (5)$$

Beyond the intersection point of the two slope lines (i.e. $n > n_{ip}$), the slope changes to s_{R2} :

$$L_n(n > n_{ip}) = \Delta L + L_1 + s_{R2} \cdot \Delta o + (s_{R1} - s_{R2}) \cdot \Delta o_{ip} \quad (6)$$

Here, Δo_{ip} relates to the distance $n_{ip} - 1$ calculated from Eq. (3) and (4) (replace n by n_{ip} and Δo by Δo_{ip}).

Literature data (e.g. the TRUMPET “C” and the CORNOPEAN “D” sound spectra¹⁸) indicate that the transition between the two slopes s_{R1} and s_{R2} takes place gradually rather than abruptly in sound spectra of reed pipes. The kink between the slopes s_{R1} and s_{R2} at the position n_{ip} in Eq. (5) and (6) should thus be smoothened as follows:

$$A = \frac{1}{\Delta p} \cdot \sum_{p_1}^p ((1-a) \cdot s_{R1} + a \cdot s_{R2}) \quad \text{with} \quad a = \frac{1}{\pi} \cdot \arctan\{s_a \cdot (2^{\Delta o} - 2^{\Delta o_{ip}})\} + \frac{1}{2} \quad (7)$$

The parameter s_a describes the “steepness” of the arc tangent function at its inflection point. The gradual transition of the slopes gives a minute improvement when modelling reed pipe spectra, where it is set constantly to $s_a = 5$ resulting in 95% of the slope transition within $n_{ip} \pm 1.2$. For flue pipes s_a is set fix at $s_a = 500$, practically resulting in a kink of the slopes s_{R1} and s_{R2} at the point of their intersect at partial number n_{ip} , because the kink represents the shape of flue pipe spectra better. The difference in round brackets corresponds to $n - n_{ip}$ (see Eq. (4)) and ensures that the transition between the two slopes s_{R1} and s_{R2} takes place symmetrically around n_{ip} . With the aid of the generalized slope parameter A one can rewrite the SPL L_n of Eq. (5) and (6) as:

$$L_n = \Delta L_G + \begin{cases} L_1 + A \cdot \Delta o, & \text{if } n \text{ is odd and } n \leq n_{ip} \\ \Delta L + L_1 + A \cdot \Delta o, & \text{else} \end{cases} \quad (8)$$

with an additional corrective term ΔL_G explained further below.

Table 4. List of all parameters used to describe the sound spectra model.

Symbol	Name	Restrains
f_1, f_n	Frequency of the first and n-th partial, respectively	
n	Partial number	$n \in \mathbb{N}$
$L_0(f)$	Frequency-dependent background (noise)	$L_0(f) < L_1$
L_1	Sound pressure level of the first partial ($n = 1$)	
p_1, p_n	Pitch category of the first and n-th partial, respectively	
$\Delta p, \Delta o$	Pitch difference in semitones and octaves, respectively	$\Delta p \geq 0, \Delta o \geq 0$
s_{R1}, s_{R2}	Slopes of the envelope function in range 1 and range 2, respectively	$s_{R2} \leq 0$
n_{ip}, o_{ip}	Transition between range 1 and range 2 expressed as partial number and as octave, respectively	$n_{ip} \geq 2, o_{ip} \geq 1$
ΔL	Sound pressure level offset for even-numbered partials in range 1	
A	Generalized slope parameter	
a	Arc tangent function providing a transition between the slopes (Eq. 7)	
s_a	Steepness of the arcus tangent functions in Eq. 7.	$s_a = 5$ for reed pipes, $s_a = 500$ for flue pipes
ΔL_G	Gaussian minimum around partial number n_G	$\Delta L_G \leq 0$
L_G	Pre-factor of the Gaussian function	$L_G \leq 0$
n_G	Position of the Gaussian function	$n_G \geq 2$ for open pipes, $n_G \geq 1$ for stopped pipes
σ	Standard deviation of the Gaussian function	$\sigma = 0.5; \sigma = 0.1$, if $n_G = 1$
c	Spectral centroid of the sound spectrum	
s	Weighted average slope of the sound spectrum	
Up to seven free parameters are used for numerical refinement of the model: $L_1, s_{R1}, s_{R2}, n_{ip}, \Delta L, L_G, n_G$		

As the diameter-to-length ratio of the resonator of an organ pipe is not infinitely small, its acoustic length is greater than that of the physical resonator (with the additional length named end correction). As a consequence, only the frequency of the fundamental, f_1 , coincides with the lowest eigenfrequency of the resonator, whereas the rest of the equidistant harmonic pipe spectrum at frequencies f_n (for $n > 1$) gradually deviates from the

eigenfrequencies of the resonator, as those are not equally spaced¹⁶. This causes minima in the harmonic SPL L_n , whose position (measured in partial number n) depend on the diameter-to-length ratio of resonator (known as pipe-scale). In published sound spectra with varying pipe-scale, the first minima are located at $n_G \approx 3$ (for the NACHTHORN¹⁷ and for an OPEN FLUTE pipe¹⁶), $n_G \approx 5$ (PRINZIPAL¹⁷), $n_G \approx 6$ (OPEN DIAPASON¹⁶), $n_G \approx 8$ (SALICIONAL¹⁶), and $n_G \approx 9$ (ZARTGEIGE¹⁷). The influence of this first minimum is modelled by a Gaussian function (Eq. (9)) with $L_G \leq 0$ and its standard deviation set to $\sigma = 0.5$ meaning that its effect is mostly restricted to the selected partial. For example, the value $\Delta L_G = 0.05 \cdot L_G$ occurs at $|n - n_G| = 1.2$, i.e. 95% of this Gaussian-shaped minimum are confined within the range of ± 1.2 partials (i.e. $n_G \pm 1.2$). The Gaussian function

$$\Delta L_G = L_G \cdot \exp\left(\frac{(n-n_G)^2}{2\sigma^2}\right) \quad (9)$$

is also used here to describe the irregular behavior of a few low-pitch reed pipes (e.g. the lowest octave of TROMBONE 16') or that of stopped pipes or semi-stopped pipes (like a CHIMNEY FLUTE) by extending the range of n_G from $n_G \geq 2$ to $n_G \geq 1$. In very few cases, where the SPL of the fundamental was very weak (as in reed pipes with half-length resonators for the lowest notes, e.g. the bass octave of CONTRA FAGOTT 16'), the position of the minimum was shifted to $n_G = 1$ and – only in this case – the standard deviation was confined to $\sigma = 0.1$.

The complete model for the envelope functions of the LTAS is given by Eq. (2), Eq. (3), Eq. (7), Eq. (8) and Eq. (9); its parameters and their restraints are comprised in Table 4.

The partial number, n , the position of the intersection point, n_{ip} , and the location of the Gaussian function, n_G , may be expressed as multiples of the fundamental frequency, f_1 , or – alternatively – in semitones above the fundamental (Δp in Eq. (4)). Since the eleventh partial ($n = 11$) lies almost exactly in the middle between two semitone steps, one may skip the procedure of rounding to integers in Eq. (9).

B. APPLICABILITY OF THIS MODEL

The model described above facilitates a simplified description of LTAS and works reasonably for the majority of ranks of organ pipes. Its seven free parameters can be extracted in the pitch range $C_0 \dots C_9$. The highest pipes may not contain enough partials any more to determine all seven parameters. Problems also occur when trying to fit the model to LTAS of pipes, which enhance certain partials or groups of them (formants), like QUINTADENA or VOX HUMANA pipes³³. CHIMNEY FLUTES, equipped with numerically optimized chimney dimensions to amplify certain partials⁵, cause significant discrepancies, because the 3rd, 4th or 5th partial is strongly amplified. One could introduce an additional term for such ranks into the model on the cost to deal with more free parameters in the numerical refinement. However, the model manages to fit the LTAS of the non-optimized CHIMNEY FLUTE pipe⁵ quite well, because its LTAS looks similar to that of a regular STOPPED FLUTE.

C. NUMERICAL DATA REFINEMENT

The criterion for the numerical least squares' refinements, i.e. that the envelope function model matches the sound spectrum of a single pipe as well as possible, is to minimize the following weighted sum:

$$\sum_n \left(\frac{1}{n} \cdot (L'_n - L_n) \right)^2 \rightarrow \min. \quad (10)$$

The weighting factor $1/n$ emphasizes the lower partials compared to the higher ones. This helps to obtain better agreement of the model for low partial numbers, where the SPL in the sound spectrum are usually greater than at high partial numbers.

The model of the envelope function (Section 3.1) was fitted to each sound spectrum of a rank yielding values for seven free parameters (Table 4, Table 6). For some spectra (e.g. the highest notes of FLUTES) the amount of partials above the noise background was too small to refine all seven of them; in such cases some parameters were kept constant. As the sound spectra were recorded in major third intervals, a rank extending over Δk octaves yielded $3 \cdot \Delta k + 1$ spectra and thus up to the same number of pitch-dependent values for each free parameter.

In a second step, the seven parameters were plotted over pitch to recognize and eventually remove outliers. Such outliers appeared, when either a sound spectrum within a rank did not quite match its neighbors, or when otherwise free parameters had to be fixed (e.g. at very high pitch). In total, about 7% of the data points were removed (Table 5). The rest of these data sets were fitted by least squares' refinement with one straight line or

two straight lines to approach these dependencies (cf. the inset figure in Table 6). These (piecewise) linear models express the dependence of $L_I(p)$, $s_{RI}(p)$, $\Delta L(p)$, $s_{R2}(p)$, $\Delta o_{ip}(p)$, $\Delta L_G(p)$ and $n_G(p)$ on the pitch category p . The major fraction of ranks allows extrapolating these functions even beyond the measured pitch range. The extrapolated pitch range was limited as soon as the parameters reached unrealistic values outside their restraints given in Table 4. These ranges were identified empirically and should represent meaningful values.

Table 5. Analyzed data.

Data	Ranks	Spectra	L_n values	Number of data points to model ranks	Outliers
Literature data	0	81	1142	0	–
StA sample set	25	366	4970	$7 \cdot 366 - 165 = 2397$	6.4%
P450 sample set	10	217	3321	$7 \cdot 217 - 130 = 1389$	8.6%

D. CATEGORIZATION OF THE SOUND SPECTRA

A study on the sound spectra of comparable registrations (a chorus of PRINCIPAL ranks and MIXTURES) of 14 different German organs concluded that 98% of their sound differences are sufficiently classified by using only three parameters. The first “is largely determined by the overall level of the organ sounds”; the second “is highly correlated with the slope of the [sound] spectra”; the third “compares the centre octave with the lowest and highest ones”³⁴. The first parameter, approximated by the total SPL of a sound spectrum is obtained by:

$$L = 20\text{dB} \cdot \log(\sum_n 10^{(L_n/20\text{dB})}) \quad (11)$$

assuming that the harmonic partials are in phase with each other and neglecting the background noise level $L_0(f)$. Alternatively, one could calculate A-weighted levels or loudness values, as they relate closer to dynamic level judgements of listeners to pipe organ sounds than the unweighted levels L do^{35,36}. Organ builders regulate the sound of the instrument in its final location by not only judging loudness but also tonal timbre³⁷, both psychoacoustic quantities. This work focuses on the two parameters for tonal timbre.

As a measure for the last of the three above-mentioned parameters, one needs the frequency of the spectral centroid c of each sound spectrum, because it is a suitable predictor for the position of the center frequency of a complex tone³⁸. The spectral centroid is commonly associated with the timbral quality “sharpness” (also referred to as “brightness”). To compare the timbre of the steady-state pipe organ tone, the tonal quality “sharpness” (on a scale from “dull” to “sharp”) has been found most suitable^{8,39}. “Sharp” and “bright” are the best verbal descriptors in English for this tonal quality, because of their good common understanding⁹. Note that “sharp” appears here as attribute for tonal timbre, not to be confused with its other meaning to describe and compare pitch. The sharpness of a complex tone is given by Eq. (12) for a modelled sound spectrum and depends on both the rank and the pitch; its value equals unity for a pure sinusoidal tone and $c > 1$ for complex tones.

$$c = \sum_n (f_n \cdot 10^{(L_n/10\text{dB})}) / \sum_n 10^{(L_n/10\text{dB})} \quad (12)$$

The “slope of the spectra”³⁴ is not precise enough to quantify it and requires clarification. In this work a weighted average slope s is calculated from the envelope function L_n , taking the differences between the SPL of adjacent partials, while reducing the weights of higher partials for increasing partial number n :

$$s = \sum_n \left(\frac{1}{n^q} \cdot (L_n - L_{n+1}) \cdot \left(\log_2 \left(\frac{n+1}{n} \right) \text{octaves} \right)^{-1} \right) / \sum_n \left(\frac{1}{n^q} \right) \quad \text{with the unit “dB/octave”} \quad (13)$$

If the exponent q equals unity, the weights equal the inverse partial numbers, n^{-1} , and the denominator is the harmonic series, which grows towards infinity. Although one cannot measure the SPL of infinitely many partials, a model should not diverge. However, for exponents $q > 1$ the denominator represents the generalized harmonic series (also: “hyperharmonic series”) converging to finite positive values given by Riemann’s Zeta function $\zeta(q)$. An appropriate value for q follows from the idea to halve the weights of each octave above the first partial while

the number of partials in them increases. This corresponds to the geometric series $\sum 2^{-\Delta o} = 1 + 1/2 + 1/4 + \dots = 2$. Comparing these values to the corresponding partial sums of the hyperharmonic series yields $q \approx 1.729$ as exponent in Eq. (13); for this q the limit of $\sum n^{-q}$ equals $\zeta(q) = 2$.

The weighted average slope s defined in this way might match the tonal quality “tension” (on a scale from “loose” to “tight”) describing the steady part of the pipe organ sounds⁸. Other adjectives from that study, albeit not associated to the category “tension”, which seem suitable are the antonyms “flutey – stringy” (for flue ranks) and “horn like – reedy” (for reed ranks). The scale “dark – bright” also appears³⁹, but involves the problem that “bright” must not be confused with “sharp” (as descriptor for c). Another more general possibility, which does not require distinction between flue and reed ranks, is “more fundamental – balanced – less fundamental”, where “balanced” suits both, DIAPASON ranks and Chorus Reeds⁴⁰.

Note that the slope of the sound spectrum, s , may become a positive value for sound spectra in which the SPL initially increase and decrease for higher partial numbers. In such cases, the initial positive slope s_{R1} might outweigh the negative value of s_{R2} as consequence of introducing the decreasing weights n^{-q} and thereby emphasizing the influence of the lower-numbered partials.

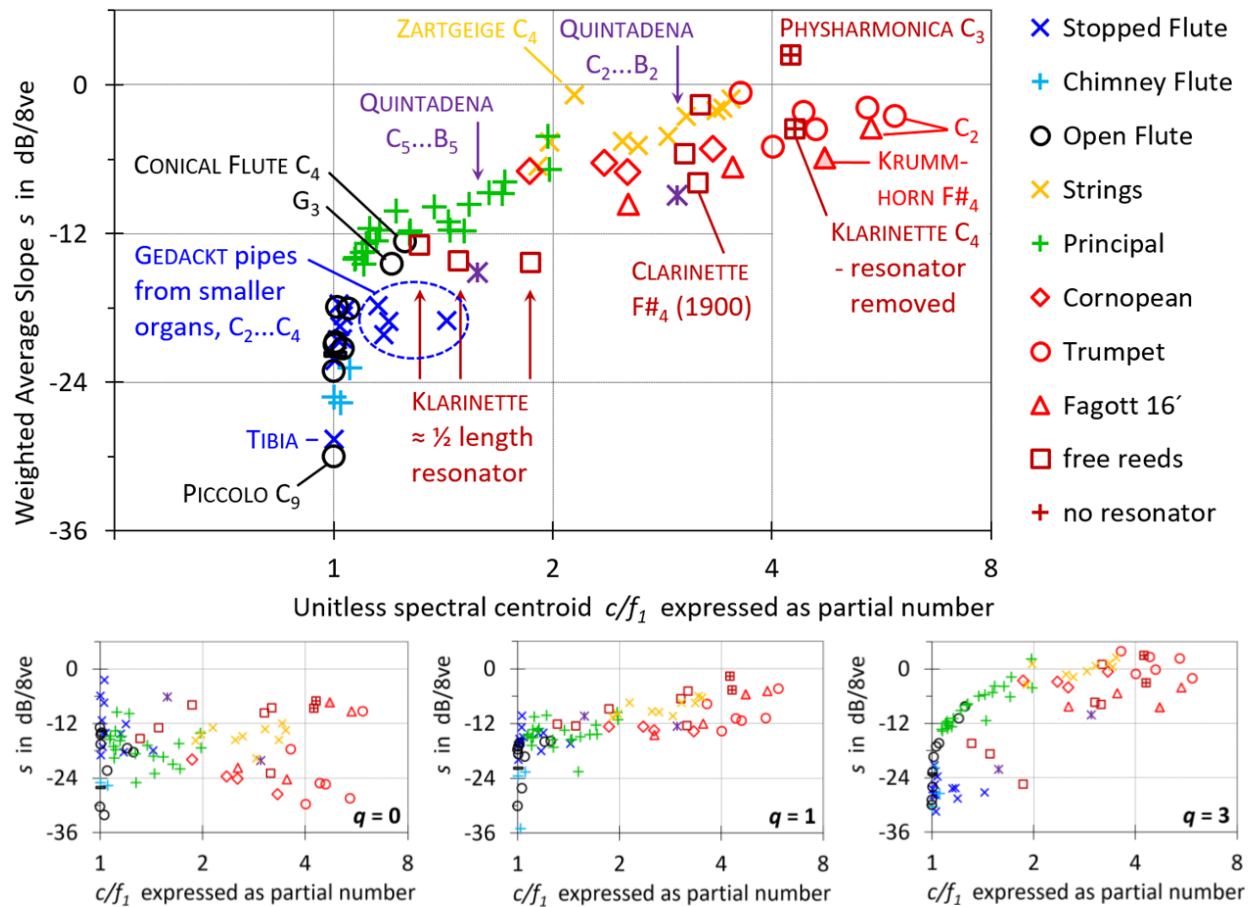


Figure 2. The literature data (Table 1) drawn for $q = 1.729$ (top) and the influence of different q values (bottom).

When plotting the weighted average slope s over the unitless spectral centroid³⁸ c/f_1 , one recognizes that different pipe organ ranks (representing different tone families) group together at distinct regions (Fig. 2, top). The exponent q influences the vertical position of the data points in this diagram, without weighing the slope s (i.e. $q = 0$) the grouping poor and too large values (i.e. $q = 3$) destroy the group of reed ranks (Fig. 2, bottom). Choosing $q = 1$ yields to a situation where the regions of DIAPASON and FLUTE ranks overlap, whereas for $q > 1.5$ they mostly separate. For values of $q > 2$ the STRING and reed ranks begin to squeeze together, as we reduce the influence of the slope at higher partials. After having observed the effect of different exponents q on the position of all the literature data points (Table 1) in the diagram, the range $1.5 < q < 2$ seems an adequate choice. This confirms the decision for the special value $q \approx 1.729$.

E. CORRELATION WITH TONE COLOR

The Tonal Director of the Schoenstein organ company, J. Bethards, has constructed an ordering scheme for organ ranks based on his long-year practical experience of pipe voicing. This diagram, called Tonal Color Wheel (TCW), is a graphical representation, in which various organ ranks are sorted according to their tonal color “as they sound in the mid-range of the manual keyboard”⁴⁰. The TCW, depicted in a very simplified and schematic version in Fig. 3, contains 63 ranks representing different timbres and classifies them more detailed than the commonly known allocation into the four organ tone families FLUTE, DIAPASON, STRING and reed ranks.

Taking the quantitative timbre values c and s derived from the 35 investigated ranks at C_4 (i.e. $p = 48$) plus those from the twelve reference sound spectra ($p = 48$ or $p = 54$) and plotting them over those angle, at which they would fit best in TCW, yields good correlations (Fig. 3, region shaded in grey excluded). Of course, one cannot expect perfect agreement, as the tonal timbre of ranks with the same name may vary considerably in different specimens. Moreover, the TCW also takes pipe shapes for the position of some stops into account. Nevertheless, flue ranks correlate well with the parameter s ($R = 0.99$ vs. $R = 0.83$ for c), while for reed ranks c is better suited ($R = -0.87$ vs. $R = -0.57$ for s). This gives strong evidence that the parameters c and s are both meaningful quantities and well suited to measure timbral properties relevant for voicing organ pipes.

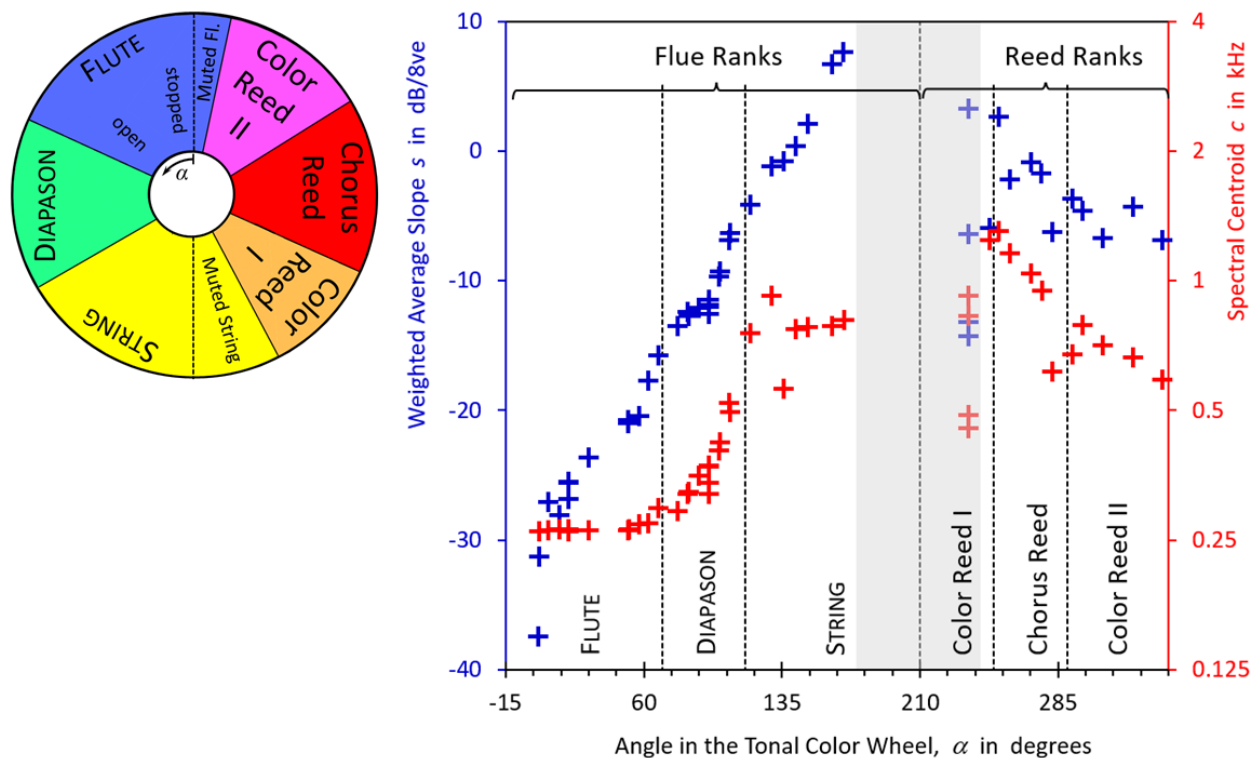


Figure 3. The Tonal Color Wheel (left) and the correlation of its angle α with timbre parameters (right).
The four excluded data points in the shaded region belong to four different CLARINET ranks.

4. RESULTS OF THE DATA REFINEMENTS

This chapter shows selected examples for pitch-dependent envelope functions and their corresponding “timbre charts”, in which the parameter s is plotted over c while the pitch is marked additionally (Fig. 4). The dotted lines in the timbre charts serve as guide to the eye and are assumed to approximately correspond to typical OPEN FLUTE⁴¹, normal DIAPASON⁴¹ and STRING ranks. The absolute position of these straight lines is estimated from literature data (Table 1), their slope is guessed from the analyzed data sets (Table 2 and Table 3) after they were plotted in timbre charts. These lines seem to correspond to the (special) case that the voicing of the pipes of a rank does not change across its whole pitch range. Table 6 gives an example for the set of the seven pitch-dependent parameters, which are described as straight lines and generate the spectra shown in Fig. 4. The parameters of all 35 investigated ranks would blow up the scope of this article and are thus compiled elsewhere⁴².

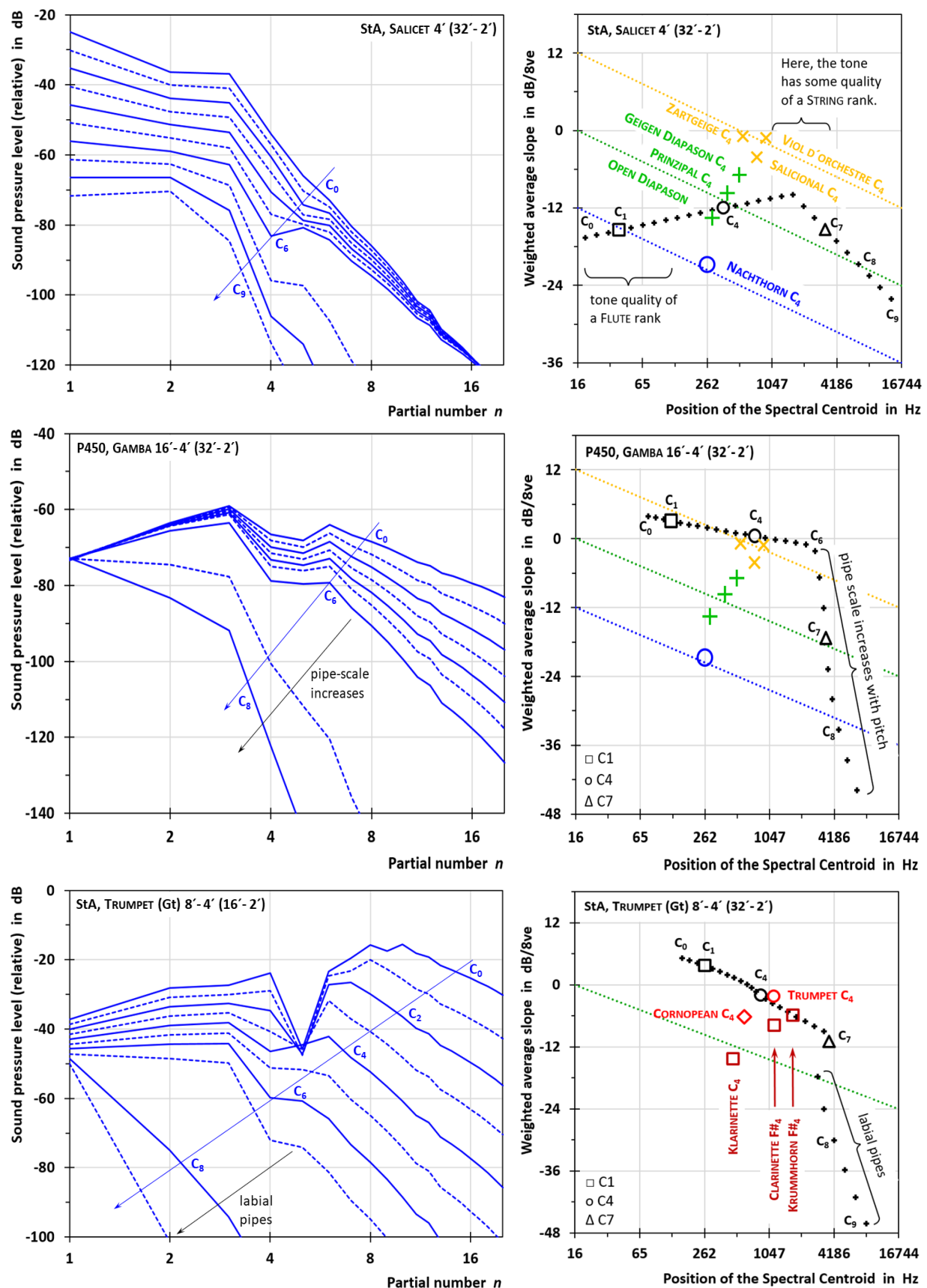
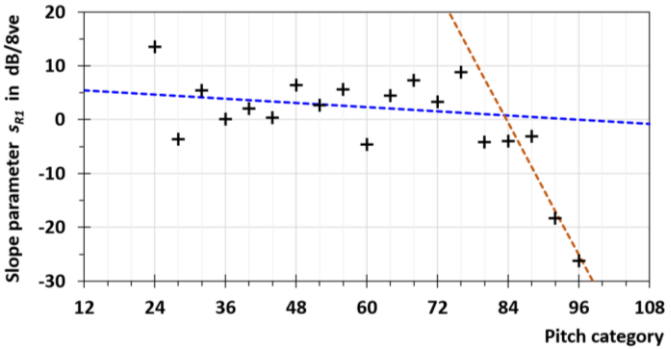


Figure 4. The LTAS of the ranks SALICET 4' (StA), GAMBA 16'-4' (P450) and TRUMPET 8'-4' (Gt, StA) plotted as envelope functions (left column) and their corresponding timbre charts (right column).

Table 6. Pitch-dependent LTAS parameters for the TRUMPET 8'-4' (Gt, StA) with linearized slope s_{R1} .

extrapolated pitch range where the model is valid: $p = 0 \dots 112$	
$L_I(p) = (-0.12 \cdot p - 37) \text{ dB}$	
$s_{R1}(p) = (-0.06 \cdot p + 6.2) \text{ dB/8ve}$ for $p < 83.3^{(*)}$	
$s_{R1}(p) = (-2.0 \cdot p + 171) \text{ dB/8ve}$ for $p > 83.3^{(*)}$	
$\Delta L = (-0.04 \cdot p + 3) \text{ dB}$	
$s_{R2}(p) = (-0.22 \cdot p - 14.5) \text{ dB/8ve}$	
$12 \cdot \Delta o_{ip} = (-0.23 \cdot p + 39) \text{ semitones}$	
$\Delta L_G(p) = (0.16 \cdot p - 24) \text{ dB}$	
$ng = -0.014p + 5.1$	
Constant values: $s_a = 5$ for reed pipes, $s_a = 500$ for flue pipes, $\sigma = 0.5$ The absolute value of L_I is arbitrary as it depends on the distance from the microphones to the pipe. *) Here, two cases appear, because the reed pipes end at $p = 83$ (B ₆) and flue pipes continue from C ₇ upwards. Therefor a separate straight line has been fitted to the top octave (brown dashed line in the inset Figure).	

5. CONCLUSIONS AND OUTLOOK

An empirical model has been developed and applied to the LTAS of various organ ranks. It allows for interpolation and extrapolation of the spectra beyond their recorded pitch range. The quality of the model is sufficient to serve as input for the calculation of psychoacoustic quantities like the pitch salience of organ tones and chords thereof⁴¹. The approach might be transferrable to the LTAS of other wind-blown musical instruments.

This study confirms that – apart from the total SPL – two parameters suffice to classify the timbre of the organ tone: the weighted average slope of the LTAS and its spectral centroid. Organ builders aimed for smooth changes of these quantities upon pitch and avoided sudden jumps, and this holds certainly true nowadays.

The fact that the weighted average slope and the spectral centroid are meaningful measures for tonal timbre implies two interesting applications. First, timbre charts could complement in planning the tonal resources (tonal colors) of the pipe organ. Secondly, as the timbre parameters loudness, weighted average slope and spectral centroid can be calculated quickly from a LTAS recorded with a good microphone, one could provide timbre charts on site. Think of a device being an extended version of an electronic tuner, which not only provides the pitch for tuning, but also displays the total SPL (or loudness) and visualizes the position of the actual pipe sound on a timbre chart. This could assist the voicer's work in the process of pipe voicing, i.e. the timbre adjustment.

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