End Corrections of Organ Pipes

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This is an experimental study of the end corrections of two rectangular wooden pipes, each wide open at one end and partly closed in various ways at the other. The pipe to be examined was excited by a telephone receiver set into one side of the pipe, and the sound was picked up through a small bore brass listening tube screwed into another side of the pipe and connected by rubber tubing to the observer's ear. The telephone receiver was fed from an audiofrequency oscillator, the frequency of which was varied to obtain the maximum resonant response. Equations for the frequency of a partly closed pipe have been suggested by several investigators. The present experiments show that not more than one of these equations is satisfactory for any extended range of apertures. The one possible exception is an equation developed by Rayleigh and by Richardson, and for this equation the present experiments are not conclusive. Incidentally attention is drawn to the restriction which a wall near an opening imposes on the flow of air through the opening, and the consequent effect on the natural frequency of an adjacent cavity. It is also found that the fundamental resonant frequency of an organ pipe may be either above or below the frequency obtained by blowing.

INTRODUCTION

ALTHOUGH the elementary theory of the vibration of air in a finite pipe of uniform cross section assumes that there is an antinode at an open end, it is well known that this is not accurately the case, and that the vibration well inside the pipe goes on as if there were an antinode a short distance outside the pipe, the distance from the end of the pipe to the position of the effective antinode being known as an "end correction." It is also known that constricting an opening at the end of a pipe lowers the natural frequency of vibration of the contained air, and therefore increases the corresponding end correction.

In an open organ pipe of the flue type the opening at the mouth is more constricted than that at the open end, and it has been known for many years that the end correction at the mouth is greater than that at the open end, so that the node in the fundamental mode of vibration lies below the middle of the pipe.

The magnitude of the end correction at an end which is wide open has been studied by various investigators. The end correction for an end which is partly closed—like the mouth of an organ pipe—has received less attention, and rectangular pipes seem to have been less investigated than circular ones. The present study deals entirely with rectangular pipes, and with ends that are not wide open.

About eighty years ago the famous French organ builder Cavaillé-Coll[1] stated that the sum of the end corrections at the mouth and open end of a rectangular pipe is given by twice the depth of the pipe—that is, twice the distance from the front of the pipe to the back of it.

Nine years earlier Wertheim[2] had carried out an experimental study on 625 pipes of various sizes, shapes, and materials. For rectangular pipes he arrived empirically at a result which is given by the rather curious equation

$$\alpha=g(b+d)\{1-(s/S)^{\frac{1}{2}}+(S/s)^{\frac{1}{4}}\}, \qquad (1)$$

in which α stands for the end correction at an end that is partly closed, b and d stand, respectively, for the breadth (side to side) and depth (front to back) of the pipe, S stands for the area of cross section of the pipe, and s stands for the area of the opening. g is a constant which differs somewhat for different materials and types of pipe.

In a rectangular organ pipe the motion of the air may be regarded as approximately two-dimensional, and a theoretical study of this two-dimensional vibration has been carried out by Brillouin.[3] He divided the pipe into two regions,

[1] Aristide Cavaillé-Coll, Comptes rendus **50**, 176 (1860).
[2] Guillaume Wertheim, Ann. chim. phys. **31**, 385 (1851).
[3] Marcel Brillouin, J. de phys. **5**, 569 (1906).

one up to the bottom of the upper lip, the other above that level. For each region he set up a velocity potential, and he made use of the conditions that the pressure and the longitudinal component of velocity must be the same on both sides of the bounding interface. In the plane of the mouth he assumed that the pressure is not a function of time—an assumption which can hardly be regarded as fitting the facts very accurately. To determine the natural frequencies of the pipe Brillouin found that the sum of a certain infinite series must vanish. Restricting himself to the first term in this series, and making the assumptions that the pipe is narrow, and the height of the mouth small compared with the depth of the pipe, he arrived at an equation[4] from which it is possible to obtain for an open pipe the relation

$$2d=-(L+\alpha)\tan\left[\pi(L-h)/(L+\alpha)\right]\times\tanh(\pi h/2d), \tag{2}$$

in which L stands for the length of the pipe—including the end correction at the top of the pipe—h for the height of the mouth, and α and d have the same meanings as in (1).

Rayleigh[5] and Richardson[6] have also obtained a theoretical equation for the natural frequencies of a tube that is closed at one end and partly closed at the other. This equation is

$$\tan kL=c/kS, \tag{3}$$

where k stands for 2π/(wave-length), L for the length of the pipe, S for the area of cross section of the pipe, and c for the conductivity of the opening at the end which is partly closed. Richardson applied the equation experimentally to tubes of circular section—each tube closed at one end, and at the other partly closed by a disk with a concentric circular hole—and found a fair agreement between the observed and calculated values.

Equation (3) is for a pipe that is closed at one end and partly closed at the other. It can readily be shown that the corresponding equation for a pipe that is wide open at one end and partly closed at the other is

$$\tan kL+kS/c=0, \tag{4}$$

where L now stands for the length of the tube plus the open end correction, and that an equivalent form is

$$\tan k\alpha=kS/c, \tag{5}$$

where α stands for the end correction at the partly closed end. If kS/c is sufficiently small (5) reduces approximately to

$$\alpha=S/c, \tag{6}$$

and if c/kS is sufficiently small (5) reduces approximately,[7] for the gravest mode of vibration, to

$$\alpha=(\lambda/4)\left[1-c\lambda/\pi^2S\right]. \tag{7}$$

Bate[8] has carried out experiments on pipes that were partly closed at one end, and has found his results satisfactorily expressed by the equation

$$\alpha=S/(2g(s/\pi)^{\frac{1}{2}}), \tag{8}$$

where g is a constant for a given shape and size of mouth, and the other symbols have the same meanings as before.

The equations quoted above differ considerably. It seemed desirable to apply them to various cases where the end corrections had been found experimentally, and to find out whether any one of the equations is in better agreement with experimental results than the others.

Method of Study

In a considerable proportion of the published experimental studies of end corrections the pipes have been excited by blowing rather than by simple resonance. Now it is known that the pitch obtained by blowing depends somewhat on the blowing stream itself. Moreover the presence of the nozzle from which the wind issues must in some cases lower the frequency that is to be measured. On the other hand, resonance is never sharp, and the frequency which gives the maximum resonant response is therefore somewhat difficult to determine.

[4] The first equation on p. 573 in reference 3.

[5] Lord Rayleigh, Phil. Trans. **161**, 89 (1871); Sci. Papers, Vol. 1, p. 46.

[6] E. G. Richardson, Proc. Phys. Soc. London **40**, 206 (1928).

[7] Following, for instance, the method employed by Stewart and Lindsay, *Acoustics* (Van Nostrand, 1930), p. 150.

[8] A. E. Bate, Phil. Mag. **10**, 617 (1930).

In the present work the resonance method has been employed, and it has proved possible to develop a technique by which different determinations of a resonant frequency are in very good agreement with each other.

The gravest natural frequency was determined by finding the frequency at which the pipe responded most strongly to a tone of variable pitch. The response was picked up by ear, through stethoscope binaurals and rubber tubing connected to a short piece of small bore brass tubing fitted into one side of the pipe. The pipe was excited by a headphone, set into one side of the pipe and fed from an audiofrequency oscillator. The oscillator had a capacitance of about 22 μf, variable in steps of 0.001 μf, and a wood core inductor with a fixed inductance of about 0.2 henry.

A change in frequency of the oscillator shifted the sound pattern in the room, so that for different frequencies the openings of the pipe lay at different positions in the sound field. To reduce errors that might arise from this source the oscillator was usually attenuated until the sound was scarcely audible to the unaided ear at a distance of one or two meters from the pipe.

To locate a resonant frequency the condenser was first adjusted rapidly until the pitch was close to that desired. Then a number of pairs of condenser settings were made, the settings in each pair being on opposite sides of the resonant position, and the observer passing quickly from one of the pair of settings to the other and deciding which gave the louder response. From several pairs of settings it was possible to decide on the setting that would give the maximum response. The condenser was then set at the position thus determined, and the frequency was found by beats made with a tone of known frequency.

For all frequencies above 112$\sim$/sec. the known frequency was obtained from a set of adjustable tuning forks. For lower frequencies a reed organ was used, the frequencies of the reeds in question having been already found by beats between their overtones and tones from the set of adjustable tuning forks.

The sum of the two end corrections was obtained by subtracting the length of the pipe from half the wave-length, the latter being found from the known speed of sound and the experimentally determined frequency. To find the end correction at the wide open end the other end was tightly closed and the resonant frequency was again determined. Assuming that the open end correction is not affected by changes at the other end, the end correction at a partly closed end was obtained by subtracting the open end correction from the sum of the end corrections. The slight enlargement in the cross section of the pipe where the exciting headphone and the brass listening tube were attached must have some slight effect on the resonant pitch and therefore on the end correction. But this effect is small. The volume of the opening for the headphone was less than 1/2000 of the volume of the pipe, and when the pipe was closed tightly at one end and left wide open at the other the end correction for the open end was in fair agreement with Wertheim's equation $\alpha = g(b+d)$.

Most of the work now reported was done with one or the other of two rectangular wooden tubes, each about 90 cm long, 9 cm$\times$9 cm inside, and with walls about 12 mm thick. Both tubes were wide open at one end and provided at the other with adjustable openings of different sizes. The work was done during the summer, and the results have been reduced to 25°C.

Height of Mouth

An increase in the height of the mouth of an organ pipe decreases the constriction at the mouth, and consequently raises the free frequency and decreases the mouth end correction. Figure 1 illustrates these effects. It shows that an extremely small opening at a closed end produces a very considerable change in pitch, and that after the opening is made a gradual increase in its size produces a less and less rapid rise in pitch. In the present case the first half-millimeter of mouth raised the pitch approximately five-twelfths of an octave, whereas 170 times that height of mouth was needed to raise the pitch a whole octave. Corresponding to this first rapid rise in pitch there is at first a rapid drop in the end correction.[9]

[9] If there is objection to speaking of a "correction" which in some cases is of the same order of magnitude as the quantity—length of pipe—to which the correction is applied, the reader may replace the term "end correction"

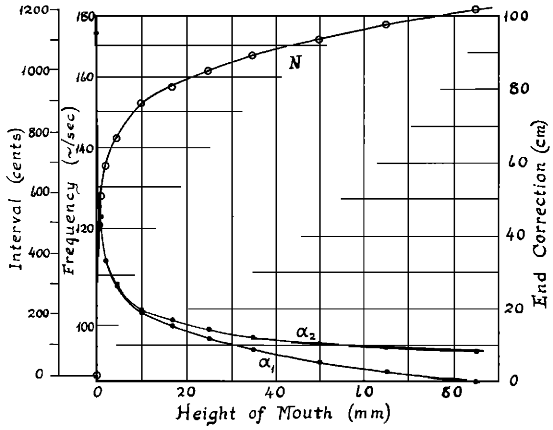

FIG. 1. Frequency (N) and mouth end correction (α) as dependent on height of mouth. For α_1 the length of the pipe was taken as the full interior length, whereas for α_2 the length was measured down to the top of the upper lip. The ordinates in the leftmost column give the rise in pitch from the condition with the mouth entirely closed.

Plates at Different Angles

The open end of a wooden organ pipe is often partly covered by a lead flap that is bent when the pipe is being tuned. Such a flap was simulated by plates that could be attached at different angles at one end of the pipe. The plates were of sheet brass 1.6 mm thick, each had a width about the same as that of the interior of the pipe, and each had a length (from the edge where it joined the pipe) of about 11 cm. Figure 2 shows the lowering of pitch on attaching various plates. It will be seen that the effect produced by a plate is small until it comes down rather close to the end of the pipe.

The accuracy with which readings of resonant frequencies can be repeated may be seen from Table I. The readings involved were taken on a morning during which the temperature of the room changed by only a little more than half a degree centigrade. Determinations of the frequencies were made alternately with the plate removed and with it attached. Determinations made on different days do not, of course, show as good an agreement, but the average deviation of six determinations made with no plate attached, and on different days, is 0.23~/sec., which is about 0.13 percent.

by the words "length to be added" at the end in question. Then for the gravest mode of vibration the length of the pipe plus the lengths to be added at the two ends will give half the wave-length.

Slit of Constant Width

Resonant frequencies were determined under a number of conditions in addition to those in actual organ pipes. In one study one end of the pipe was wide open and the other was closed by a piece of 1.6-mm sheet brass in which there was a slit of constant width. This slit could be set at different distances from a wall of the pipe, as shown in Fig. 3. When such a slit is near a wall the motion of the air through the slit is more restricted, and the resonant frequency of the pipe is correspondingly lower,[10] than when the same slit is farther from the wall.

Figure 4 shows this effect. It will be seen that the narrower the slit the lower is the pitch, and that in these cases the lowering in pitch as the slit approaches the wall is approximately the same for all four slits—in the neighborhood of 80 cents.

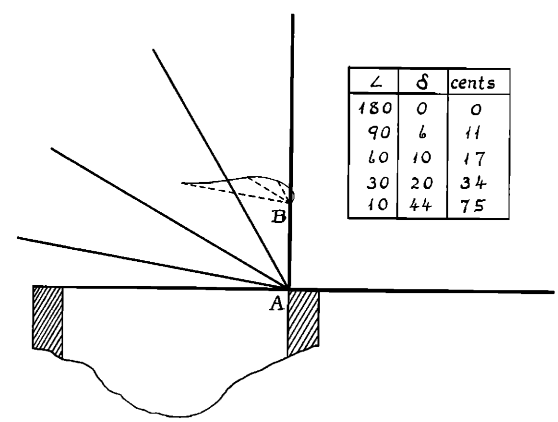

FIG. 2. Lowering of pitch produced by plates extending at different angles at one end of a pipe. The insert gives the angle (∠) between the plate and the plane of the otherwise open end, the corresponding increase (δ) in end correction, in mm, and the lowering of pitch produced by attaching the plate. The lines extending outward from A show the positions of the plates. The distance AB represents the end correction without any plate, and the lengths of the dotted lines extending outward from B represent the increments in end correction when plates in the corresponding directions are attached.

Other Cases

A number of other cases have been examined. In one study one end of the pipe was closed by two pieces of 1.6-mm sheet brass between which was a slit of variable width extending from one side of the pipe to the other. In one series of

[10] Léon Marty, Revue d'acoustique 4, 13 (1935).

observations one edge of the slit was kept flush with a wall of the pipe, and in another series the middle of the slit was kept halfway from one wall to the other. In the former case the flow of air through the slit was somewhat obstructed by the presence of the wall, and Fig. 5 shows that for moderate widths of slit the pitch is therefore somewhat lower than for a symmetrically placed slit of equal width.

In another series use was made of a smaller symmetrical rectangular opening. One dimension was maintained at 11.7 mm, and the other was varied from zero to the distance between the walls of the pipe. The changes were similar to those shown in Fig. 1, except that they were more gradual and not as great.

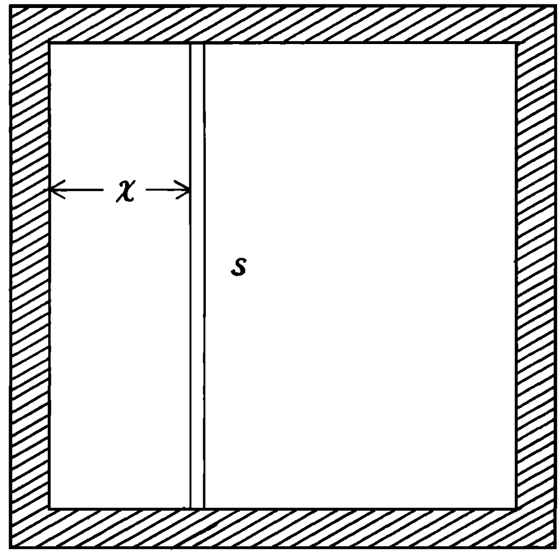

FIG. 3. Position of movable slit. We are looking out from the inside of the pipe. The slit s may be fixed at different distances x from one wall of the pipe.

Comparison of Equations

Cavaillé-Coll

From the statement quoted above from Cavaillé-Coll it follows that the fundamental resonant frequency of a pipe should be independent of the height of the mouth. This is certainly not the case. For the pipe of Fig. 1 Cavaillé-Coll's rule would be correct for a height of mouth of 17.6 mm, which is about a fifth of the depth of the pipe. This is not far from the ratio of height of mouth to depth of pipe that is actually employed for organ pipes, so that for real organ pipes Cavaillé-Coll's rule is probably sufficiently good.

Wertheim

In order to make use of Eq. (1) we must know the value of the g which appears in it. Wertheim found his results fitted best by taking $g=0.187$ in all cases of pipes that were open at both ends, whatever the material of which the pipes were made. For pipes that were closed at one end the value of g depended on the material of which the pipes were made. For wooden pipes he found $g=0.240$. For pipes that were wide open at one end and partly closed at the other he made use of the value 0.187. To see how well my experimentally found end corrections agree with those obtained from Wertheim's equation I have made calculations for several cases and with each of the two values for g. Figure 6 shows the case in which Wertheim's equation gives the best agreement with my experimental results. Even in this case the agreement is far from satisfactory. It is possible that a somewhat better agreement might have been obtained by some other choice for the value of g. I have not tried making other choices.

TABLE I. *Frequencies with plate at 90°.*

	Plate Attached	Plate Removed
		173.79
	172.61	
		173.73
	172.68	
		173.72
	172.59	
		173.71
Av.	172.63	173.74
Av. Dev.	0.04	0.03

Difference 11 cents
Additional end correction because of plate, 6.4 mm.

In order to be sure that I am not making some blunder in the use of Wertheim's equation I have carried through the calculations for one set of his own experiments. The agreement is much better than that in Fig. 6.

Brillouin

Equation (2) can be solved for α by a method of successive approximation. Since this equation was obtained for a pipe that has a mouth in the usual position—that is, in the side of the

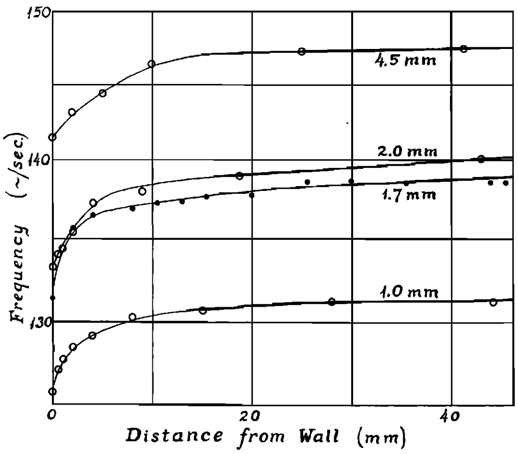

FIG. 4. Four slits of constant width at different distances from a wall of the pipe. In each case one end of the pipe was wide open and the other was closed except for the slit, and the slit was parallel to two walls as shown in Fig. 3. Abscissas are distances from a wall of the pipe to the nearer edge of the slit. The number on each curve is the width of the corresponding slit.

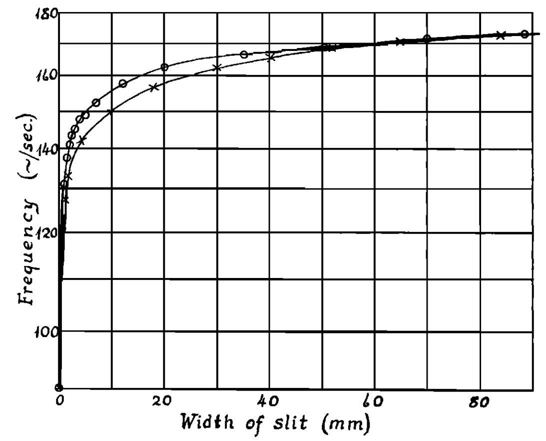

FIG. 5. Slit in end of a pipe. In the upper curve the middle of the slit was at the middle of the end of the pipe. In the lower curve one edge of the slit was flush with the inside of the wall.

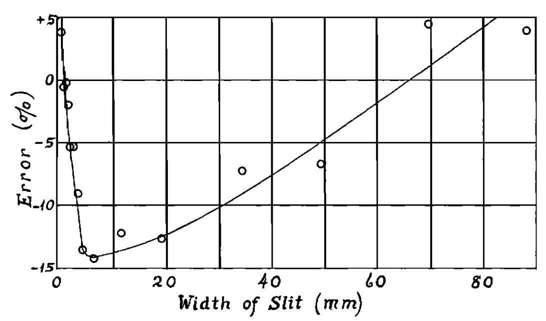

FIG. 6. Errors in end correction obtained by using Wertheim's equation. This is for the same case as the upper curve in Fig. 5. In these calculations g has been taken as 0.187.

pipe—I have applied it to only that one case. Figure 7 shows the result. For this particular pipe the error given by Brillouin's equation is least when the mouth is extremely small, and also when it has a height of some one-fourth to a fifth of the depth of the pipe. As we move away from either of these regions the errors become rapidly worse. This fact is not surprising in view of the various assumptions and approximations that were made to obtain the equation.

Rayleigh-Richardson and Bate

The Rayleigh-Richardson equation (3), or the equations obtained from it, cannot be employed to find the magnitude of an end correction unless we already know the conductivity of the opening. Rayleigh[11] showed that the conductivity of a circular hole in a large thin wall equals the diameter of the hole, and he made calculations[12] of the extent to which the conductivities of elliptic apertures of various eccentricities exceed the conductivities of circular apertures of equal area. For rectangular openings in which the ratio of the sides varied from 4 : 3 to 3 : 1 Bate[8] found experimentally that the conductivity equals that of an elliptic opening of the same area when the ratio of the sides of the rectangle equals the ratio of the axes of the ellipse. Making use of Rayleigh's calculations for elliptic apertures and Bate's relation between elliptic and rectangular apertures it is possible to calculate the conductivities of the rectangular openings that I have employed, and so to compare the experimentally found end corrections with those obtained from the Rayleigh-Richardson equation.

Instead of proceeding as just outlined, and employing the Rayleigh-Richardson equation to calculate end corrections, it seemed easier to use each experimentally found end correction and make use of Eq. (5) to calculate the corresponding conductivity, and then to compare this conductivity with that calculated from the size

[11] Lord Rayleigh, Phil. Trans. **161**, 95 (1871); *Sci. Papers* (Cambridge University Press), Vol. 1, p. 52; *Sound*, second edition (Macmillan, 1896), Vol. 2, p. 178.

[12] Lord Rayleigh, *Sound*, second edition (1896), Vol. 2, p. 179.

and shape of the opening. Figure 8 shows some of the results.[13]

For a pipe with a mouth in the side—as in most actual organ pipes—Fig. 8 shows that the agreement is far from satisfactory when the mouth is cut up much farther than it is in real organ pipes. For a symmetrically placed slit which extends from one side of the pipe to the other the agreement is again far from good—especially at the wider openings. For short openings of moderate widths the agreement is better. These results are not surprising when we recall that a conductivity calculated from size and shape assumes that the opening is in a wall which extends to a considerable distance, whereas the walls for my larger openings extend to only a short distance. In fact, Bate[8] has shown that the conductivity of an opening depends not only on the opening itself but also on the surroundings, and Robinson[14] has pointed out that measured conductivities are in some cases many times as large as those obtained theoretically. For this reason the results shown in Fig. 8 do not offer any real test of the Rayleigh-Richardson equation.

Bate's equation is essentially a particular case of the Rayleigh-Richardson equation. Bate found the g in his equation running from about 1.0 to 1.5. If we take $g=1$, and take the opening as circular, with its conductivity equal to its diameter,[11] (8) is equivalent to (6). Now (6) is obtained from (5) by assuming kS/c small enough to replace by its tangent. In my work kS runs from about 1.5 cm to 3 cm, so that a large opening would be needed to make kS/c sufficiently small to use either (6) or (8). If the Rayleigh-Richardson equation is valid it is obvious that I cannot expect any extended agreement between my results and Bate's equation. Figure 9 shows how they do compare.

Bate found that his equation fitted rather satisfactorily the experimental results that he

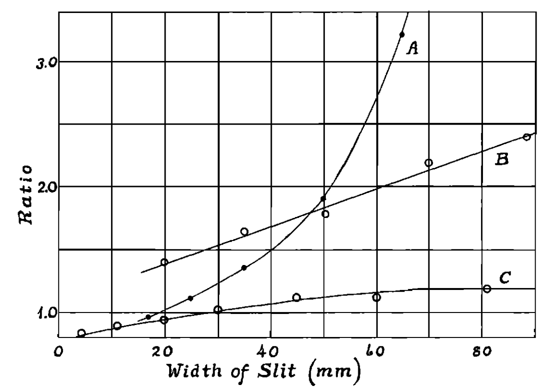

FIG. 8. Ratio of conductivities of openings calculated from the Rayleigh-Richardson equation to the conductivities calculated from size and shape. A, Mouth in side of pipe. B, symmetrical slit across end of pipe. C, shorter rectangular opening in end of pipe.

obtained, but he made use of only three heights of mouth—1.00 cm, 1.25 cm, and 1.50 cm. For two of the cases shown in Fig. 9 it will be seen that if we deal only with slits of about these same widths my experimental results will fit the equation tolerably well with a choice of g in the neighborhood of 1.3. Except in this particular region the agreement is far from good.

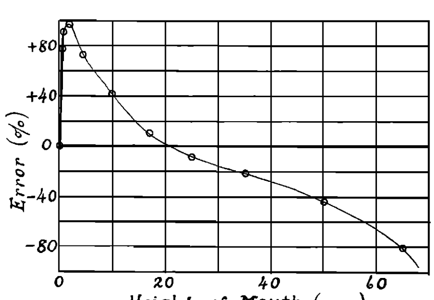

FIG. 7. Errors in end correction obtained by using Brillouin's equation.

SUMMARY

On comparing the results obtained from the various expressions for an end correction it is seen that each of the equations is in satisfactory agreement with the experimental values in some limited range, and that none of them—unless perhaps the Rayleigh-Richardson equation, for which the evidence is not conclusive—fits the experimental results over any considerable range. No equation that I have obtained empirically is much better. Nevertheless, the concept of an end correction is of considerable value, and it is to

[13] Rayleigh's table of conductivities does not include ellipses in which the ratio of the axes exceeds 6 : 1, so that without making the calculations necessary to extend his Table I cannot include in Fig. 8 any results for my narrower slits.

[14] N. W. Robinson, Proc. Phys. Soc. London **46**, 772 (1934).

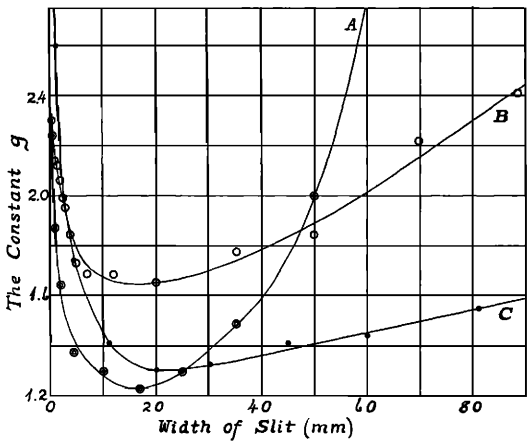

FIG. 9. The constant g in Bate's equation. Ordinates are values that g would have to take for my experimental results to be in agreement with the equation. A, mouth in side of pipe. B, symmetrical slit across end of pipe. C, shorter rectangular opening in end of pipe.

be hoped that some equation which fits experimental results satisfactorily will soon be found.

APPENDIX

In certain cases Rayleigh[15] found that the pitch of maximum resonant response of an organ pipe was somewhat lower than the pitch of the same pipe when blown. Bukofzer[16] has found that this also holds true of panpipes when they are blown mechanically. The question arose as to whether this relationship holds for organ pipes in general. Several organ builders kindly made available for a test of the matter a total of a dozen open wood pipes. Each of these was blown at the pressure for which it had been voiced, and the frequency thus obtained was compared with the fundamental resonant frequency as determined by the method described above. The slight change in pitch produced by the addition of the exciting phone and the listening tube were unimportant, because the resonant frequency and the frequency when blown were both of them determined after these had been attached.

The result is in general not in agreement with that for the pipes examined by Rayleigh. With two of the pipes now tested the resonant frequency was a trifle lower than the frequency when blown, but with the rest the resonant frequency was the higher. The difference was not great. Excluding one pipe that did not speak well, the average difference amounted to about 20 cents, and the maximum was about 50 cents. Apparently it is not the case that organ pipes when blown are always somewhat sharper than their fundamental resonant pitch, nor that they are always somewhat flatter. Whether they are sharper or flatter may depend on the voicer who does the adjusting.

In conclusion I wish to express my most hearty thanks to my colleague James F. Koehler, who designed and built the oscillator for me, and to the four organ builders who provided organ pipes: Aeolian-Skinner Organ Company; Austin Organs, Inc.; Estey Organ Corporation; The Ernest M. Skinner and Son Company, Inc.

[15] Lord Rayleigh, Phil. Mag. **3**, 462 (1877); **13**, 340 (1882); *Sound*, second edition (Macmillan, 1896), Vol. 2, p. 219.

[16] Manfred Bukofzer, Zeits. f. Physik **99**, 643 (1936).